Vessel Segmentation and Blood Flow Simulation Using Level-Sets and Embedded Boundary Methods

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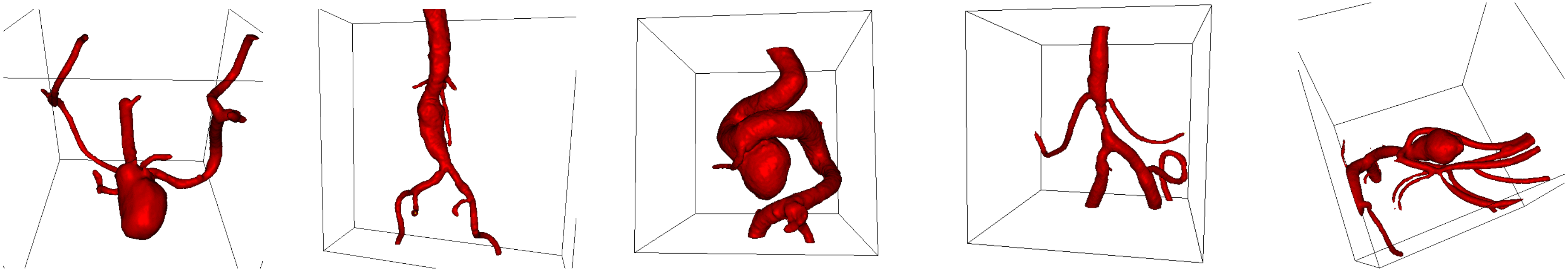


Outline

1. Introduction
2. Vessel extraction
3. Building the CFD Grid
4. Blood-Flow Simulation

Introduction

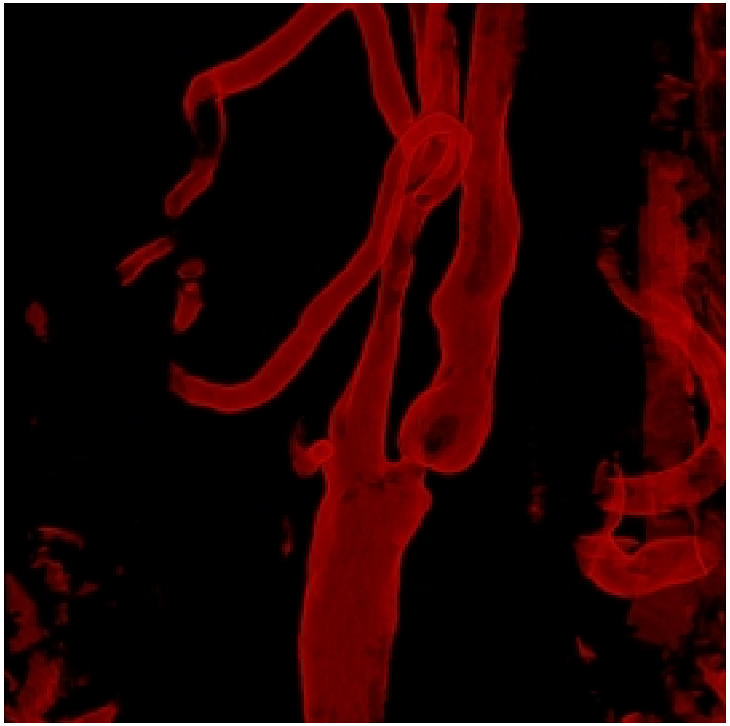
Application: Blood Flow simulation

Problem:

- Plaque, strokes
- Study of flow in pathologies

Tasks:

- Surface extraction algorithms
- Meshing the realistic geometries
- Simulating the Blood-Flow
  - wall shear stress
  - pressure
  - flow velocity & patterns

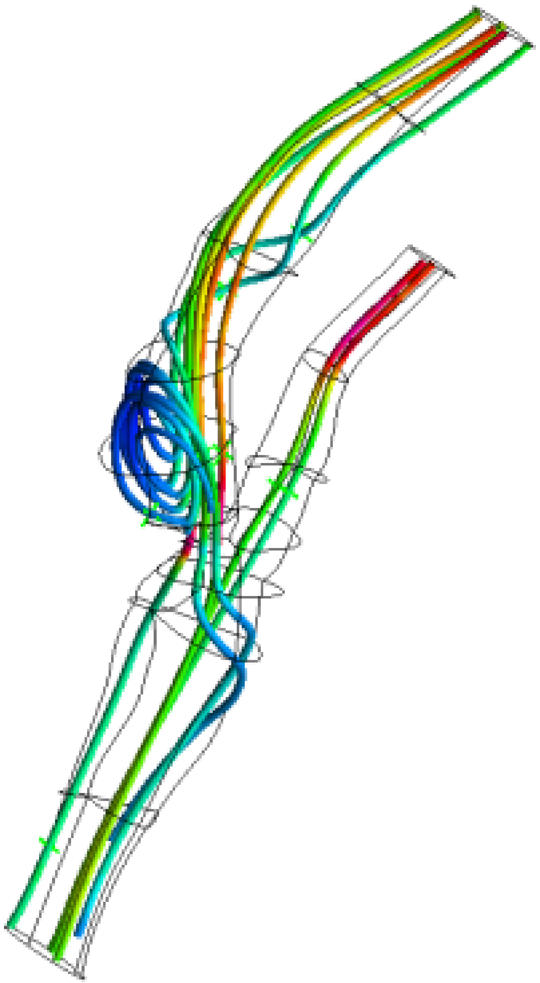


Volume Rendering (VR) image of a carotid MR dataset

From the image to the flow

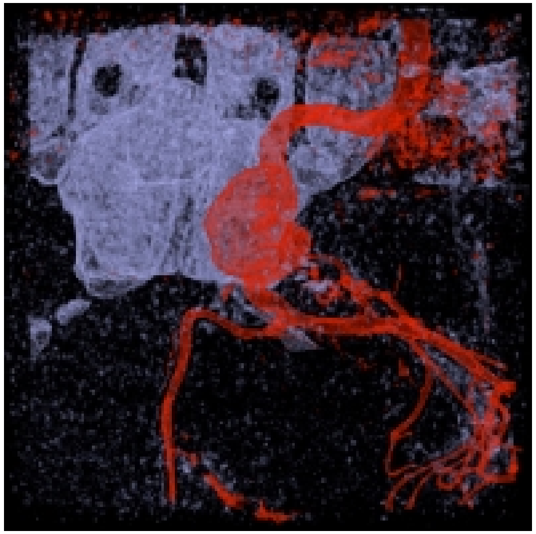
Steps:

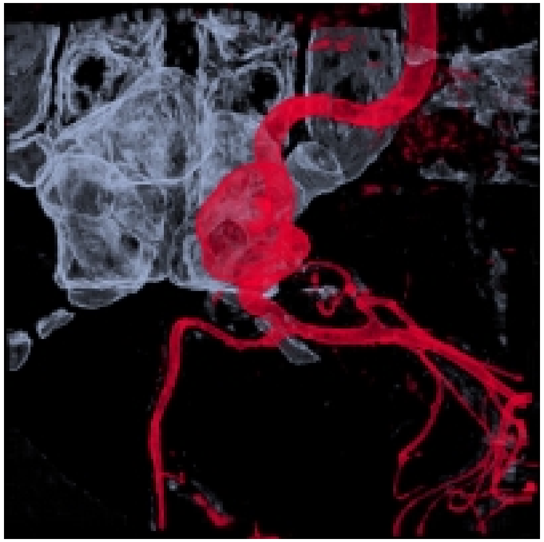
- Preprocessing (filtering) the image
- Extracting the surface from the Image
- Modifying the surface
  - with centerlines
  - with curvature motion
- Building a surface mesh
  - Nurbs patching
- Building a 3D finite-element mesh
- Simulating the blood flow in complex geometries



Surface Extraction

Pre-processing the data





The Beltrami Flow is a general model of Anisotropic diffusion.

It smoothes the data and enhances edges.

$$I_t = \frac{(1+I_y^2)I_{xx} - 2I_xI_yI_{xy} + (1+I_x^2)I_{yy}}{(1+I_x^2+I_y^2)^2}$$

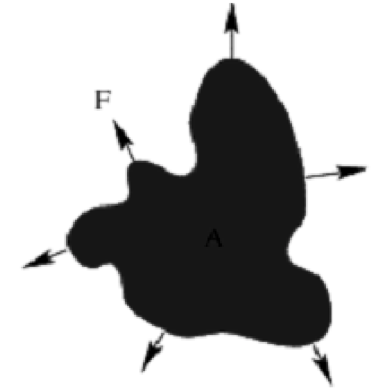
A General Framework for Low Level Vision (1998), Sochen, Kimmel, Malladi, IEEE TMI

Deschamps, Malladi, Ravve, “Fast Image Manifolds for Filtering and Segmentation of 3D Medical Images” IEEE TVCG (pending)

Surface extraction with Level-Sets

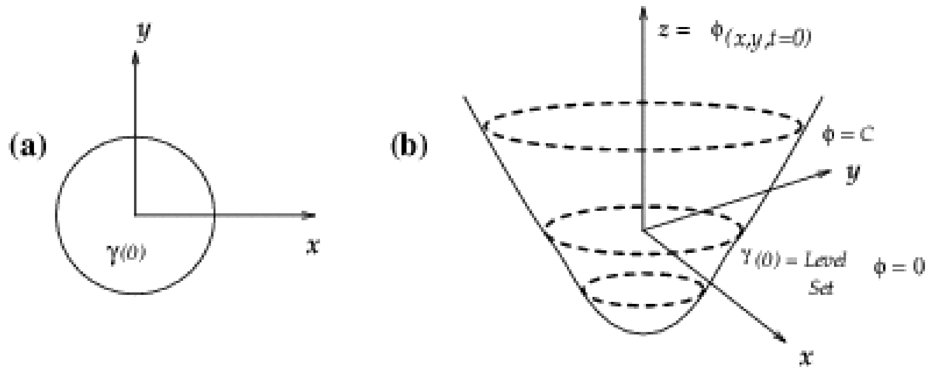
We have a contour C moving in its normal direction with speed F

$$\frac{\partial C}{\partial t} = F\vec{n}$$



The speed is chosen to trap the contour where the contrast is high in the images.

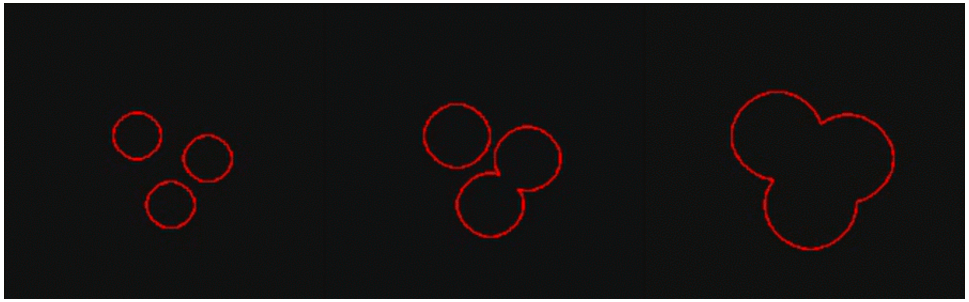
We embed it into a signed distance function.

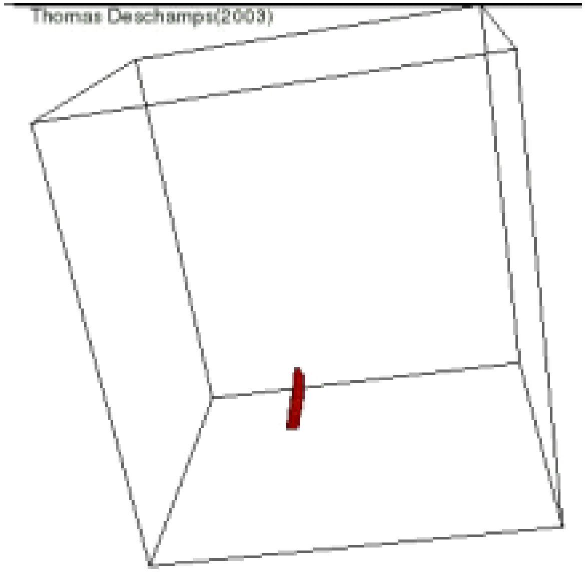


It leads to the Level Set equation: $\phi_t + F|\nabla\phi| = 0$

The contour can handle topology changes while moving.

Examples of segmentation with Level-Sets



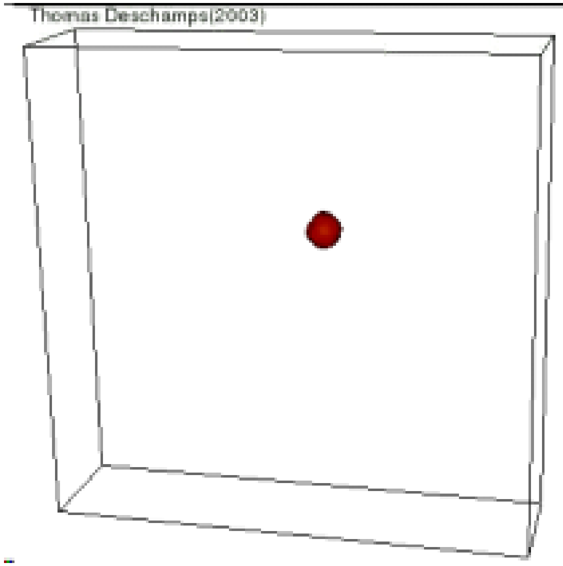


Consider a speed $F = k_I(\mathbf{x}) = e^{-\alpha|\nabla I(\mathbf{x})|}$.

It slows down where the contrast is high.

More stophisticated flows: $F = k_I(1 - \epsilon K) - \nabla k_I \cdot \vec{n}$

However, high computational cost limit their usefulness



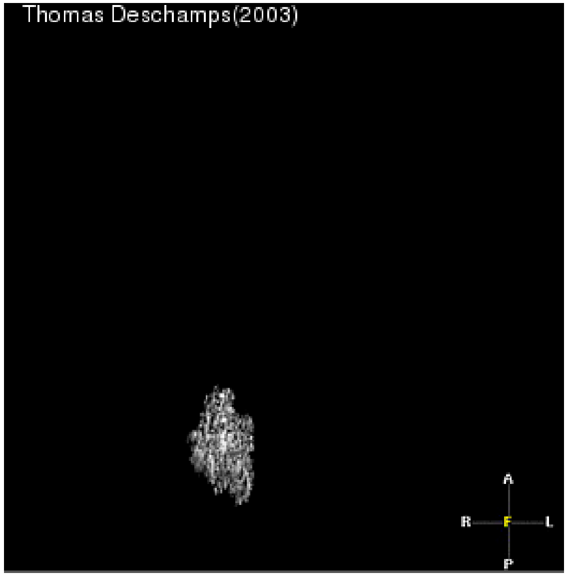
The special case of the Eikonal Equation

Monotonic speed (strictly inward/outward) leads to the Eikonal Equation.

$$F|\nabla T| = 1$$

Solved with o(n log(n)) Fast marching algorithm (J.A. Sethian)

Surface extraction: speed $F = k_I, \quad \text{where,} \quad k_I(\mathbf{x}) = e^{-\alpha|\nabla I(\mathbf{x})|}$

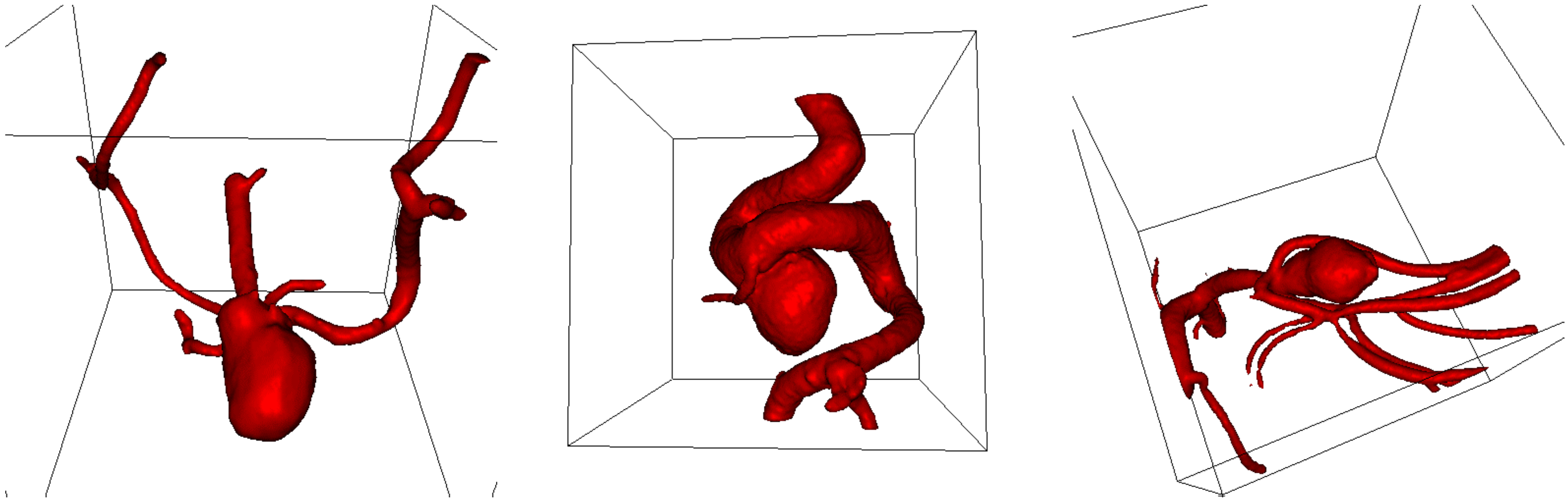


No curvature, and no stopping condition.

Used for initializing Level-Sets

Example on cerebral aneurysms

Maximum Intensity Projection (MIP) images of aneurysms



Concurrent seed points inside and outside: we extract the surface where fronts collide.

The Level-Sets method needs only a few iterations

The accuracy of the surface extraction is below the image resolution.

Building the CFD Grid

Why a CFD Grid?

In order to simulate the Blood-Flow inside the vascular object, we have to build a discretization of the domain where the Navier-Stokes equations have to be solved.

Many methods extrapolate a Finite-Element gridding of the 3D domain from the surface of the vascular object.

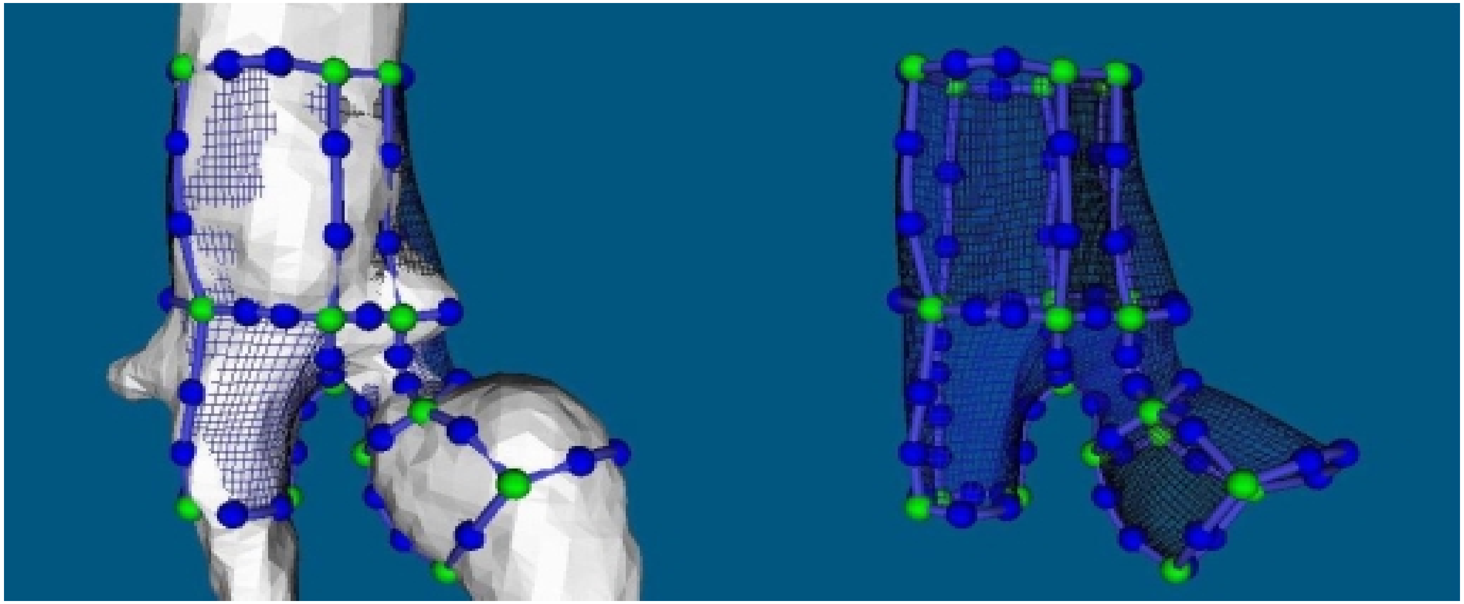
The zero-level set is embedded in the signed-distance. We could also extract it with the Marching Cubes Algorithm.

But for extrapolating the 3D grid, the surface has to be described by other geometries like NURBS, structured grids, etc…

Limitations of remeshing techniques

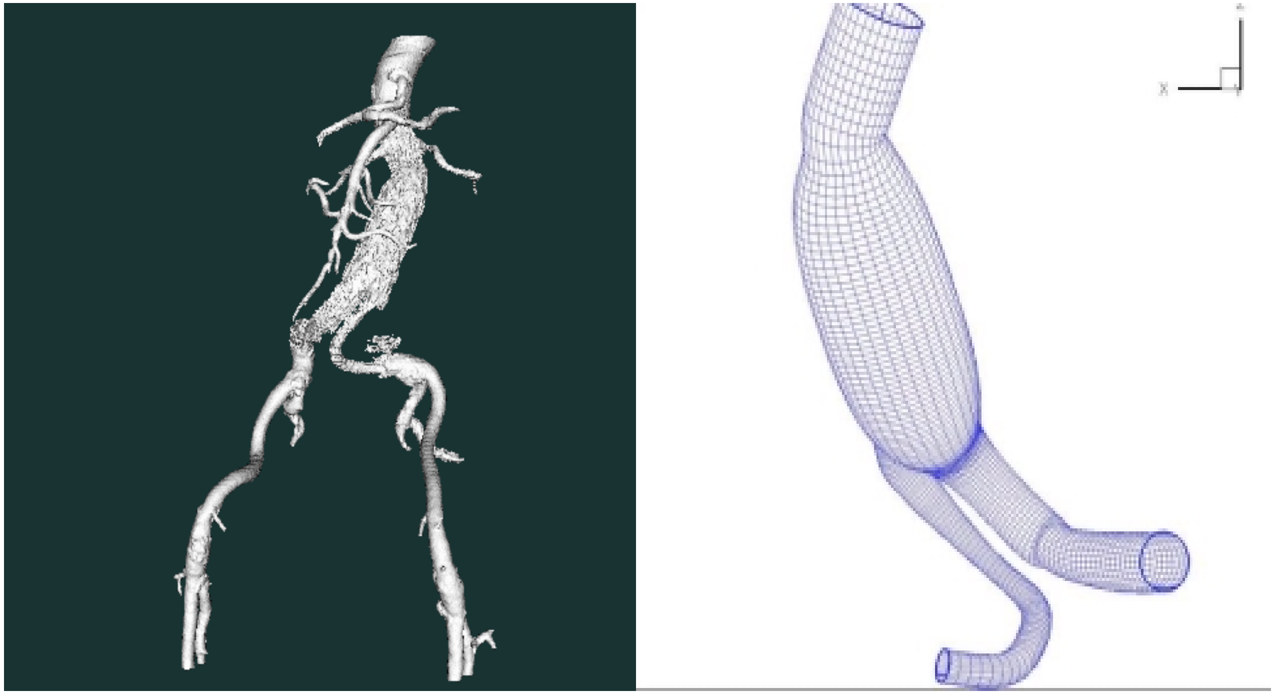
High number of patches = Lots of interaction + huge computing time

Small number of patches = poor quality of the mesh



Building the patches on the surfaces of the carotid bifurcation

Limitations of remeshing techniques



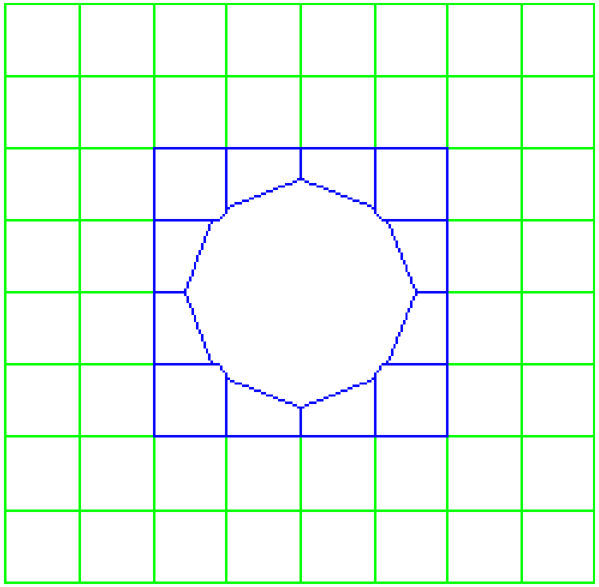
The mesh is built by

- Cutting the surface along the centerline
- Approximating the cross-section (by a circle sometimes) and
- Interpolating in between sections
- Carefully designing the bifurcations

Cartesian Grid Representation of Irregular Boundaries

The irregular geometry is now represented on a rectangular grid by specifying the intersection of each grid cell with the region on one or the other side of the boundary.

This leads to a natural finite-differences discretization of the solution to PDEs on either side of the boundary.



On the boundary, some care must be taken.

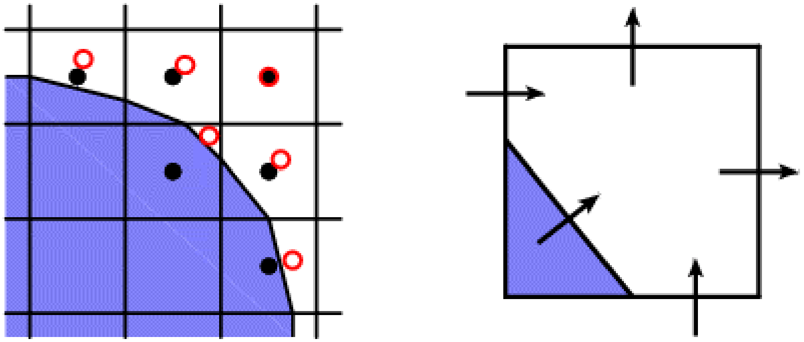
Regular rectangular structure grids

- grid generation problem is solved
- very similar to the Level-Sets

Embedded Boundary Discretization of Conservation Laws

Consider a hyperbolic system of conservation laws: $$\frac{\partial U}{\partial t} + \nabla \cdot \vec{F}(U) = 0$$

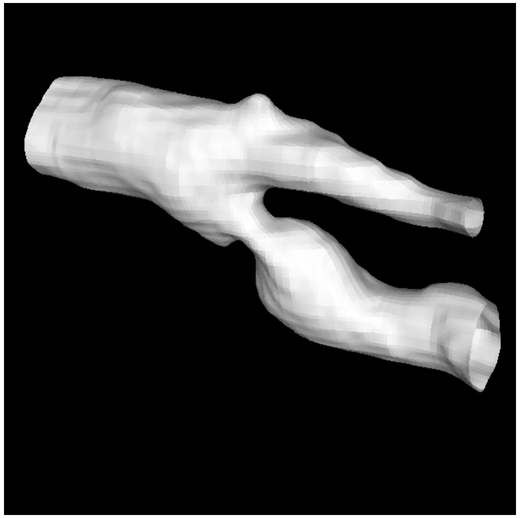
Primary dependent variables approximate values at centers of cells.



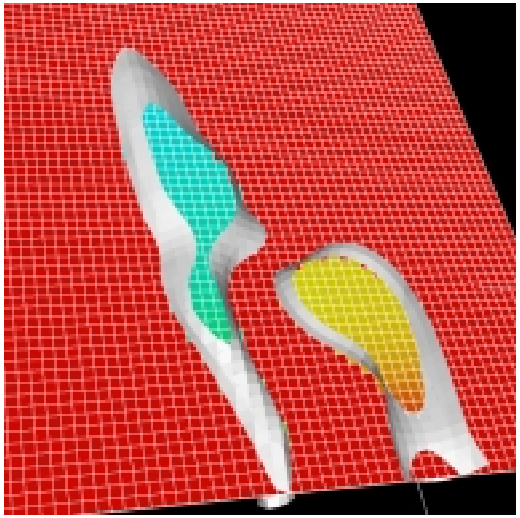
Divergence theorem over each control volume leads to finite volume approximation:

$$\nabla \cdot \vec{F} \approx \frac{1}{\kappa \Delta x^D} \int \nabla \cdot \vec{F} dx = \frac{1}{\kappa \Delta x} \sum \alpha_s \vec{F}_s \cdot \vec{n}_s + \alpha_B \vec{F} \cdot \vec{n}_B \equiv D \cdot \vec{F}^c$$

From Level-Set to Embedded Boundary



Level-Sets



Embedded-Boundary

To build the EB, one needs only to obtain the area, volumes, locations of centroids, and the average outward normal to the boundary.
Some of the quantities can be derived from the divergence theorem.

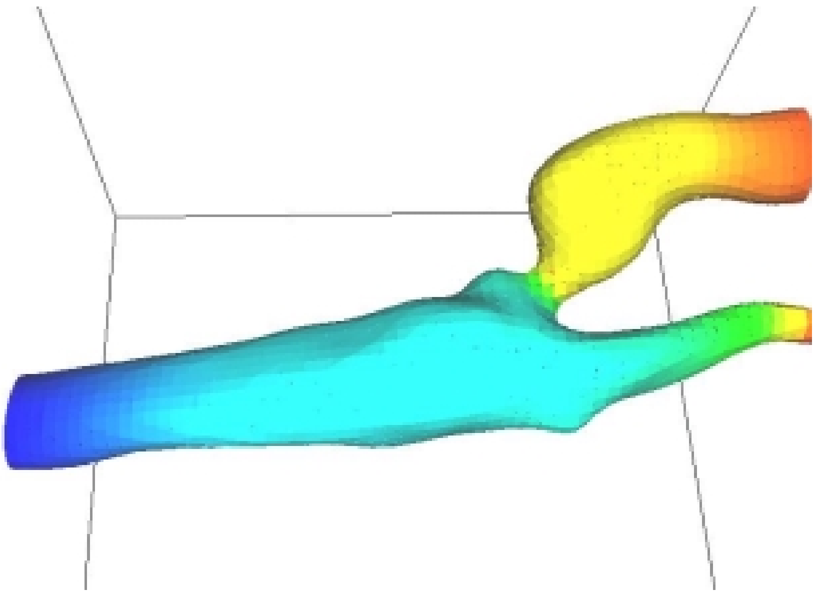
The definition of the distance function on the Cartesian grid makes this process straightforward.

Blood-Flow Simulation

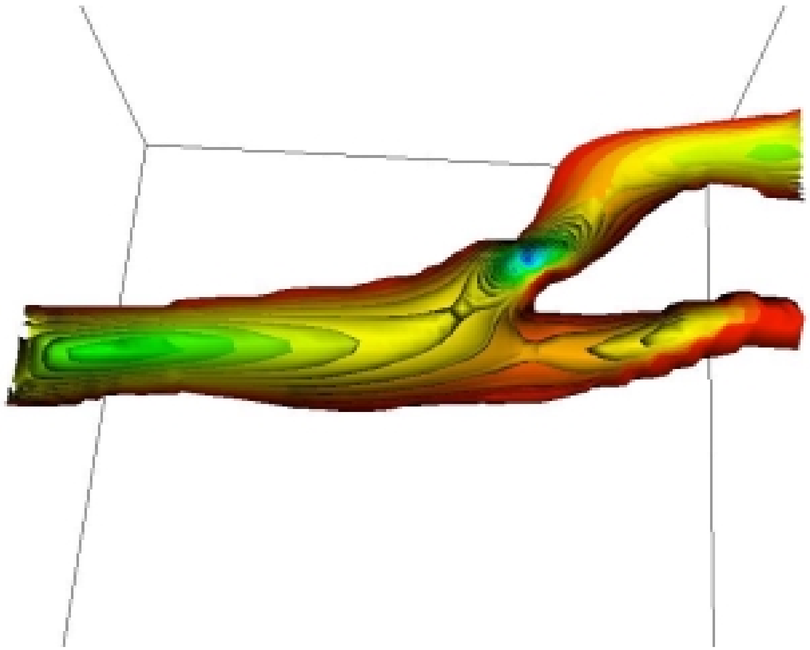
Navier-Stokes solutions

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \nabla p = \nu \Delta \vec{u}$$

$$\nabla \cdot \vec{u} = 0$$



Pressure on the surface of the carotid
High: blue, low:red



Velocity inside the carotid

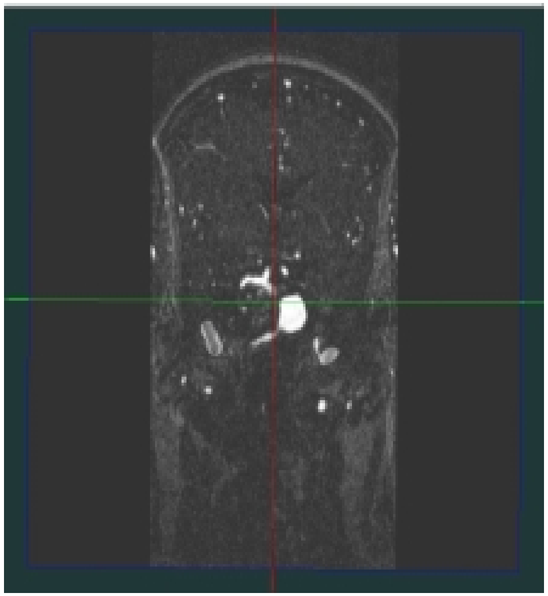
Summary

1. Surface Extraction
   1. Fast
   2. accurate

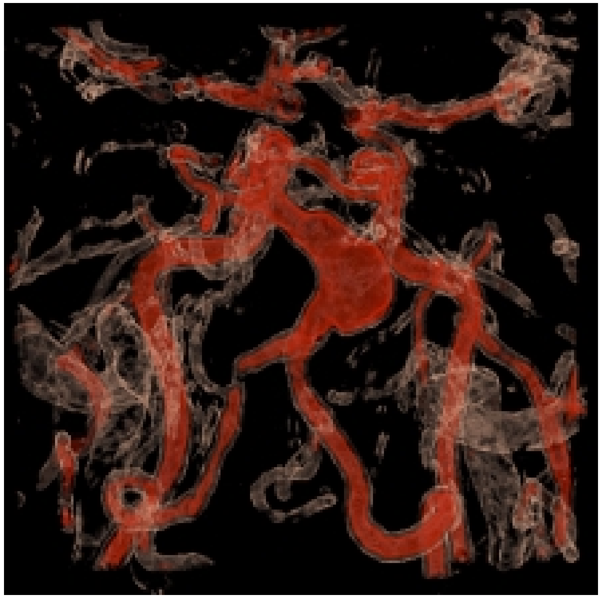
2. Embedded Boundary definition
   1. Built from level-sets automatically
   2. Gridding of the whole domain

3. Blood-Flow simulations
   1. Use of Chombo library
   2. Possibility to use AMR, multigrid, etc…

Future work: Evolution of pathologies



MR dataset



Volume Rendering

Understand:

- Relation between flow and plaque.
- Relation between flow and pathology

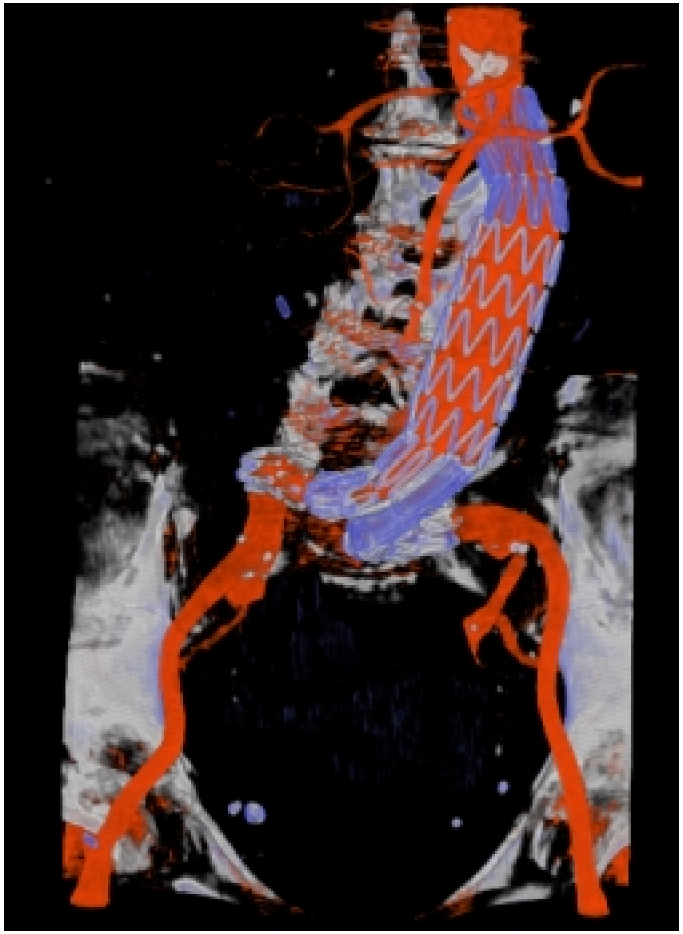
Predict:

- plaque formation
- pathology evolution

Future work

Simulation:

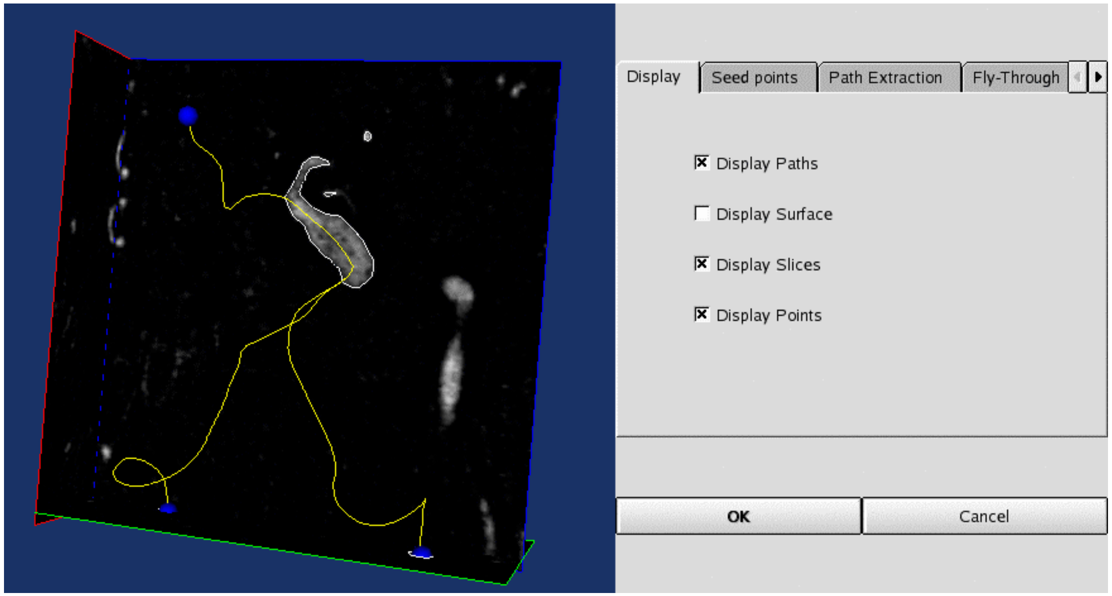
- Adaptive Mesh Refinement techniques
- Parallel implementation
- Moving boundaries

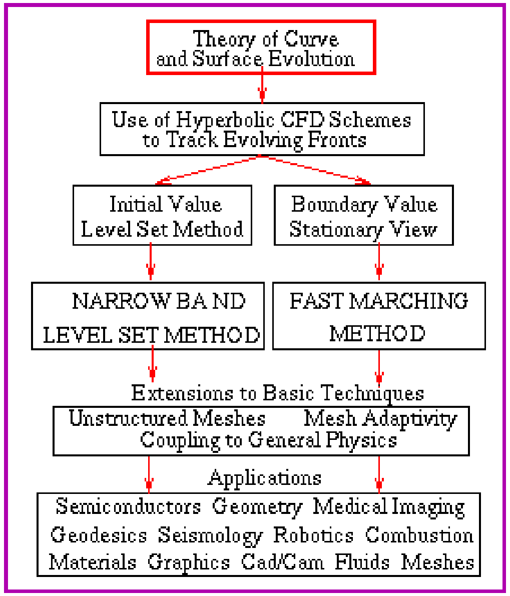


Application to other flows:

- Organs with stents (Pr Dwyer, UC Davis)
- Air Flow in the Trachea/Larynx (LLNL)
- Including plaque in the simulation

The Segmentation software package



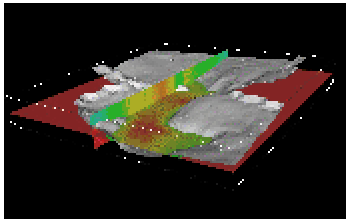


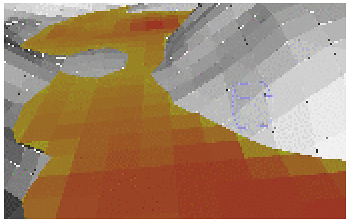
1. Used at Life-Sience Dept LBNL, UCSF VA Vascular Surgery
2. C++ implementation of Curve evolution
3. Filtering, Fast-Marching, Level-Sets, Path extraction
4. Using free software: VTK, Qt, CMake

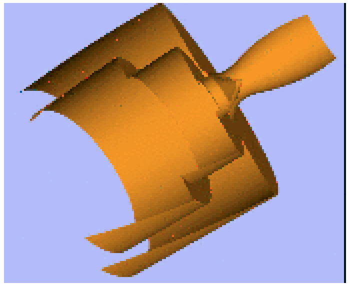
Chombo: a software framework for Volume-of-Fluid discretization methods

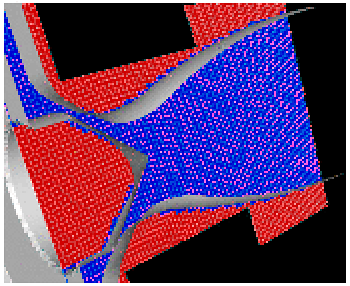
The Chombo library (http://seesar.lbl.gov/anag/chombo/) tools for the numerical solution of PDEs discretized on a Cartesian grid.

- Adaptive Mesh Refinement
- Mixed language model: C++ (data-structures), Fortran (calculations)
- Build on public-domain standards: MPI, HDF5, VTK
- Interoperability with other tools: grid generation, solvers, etc…









Vessel Segmentation and Blood Flow Simulation Using Level-Sets and Embedded Boundary Methods, T. Deschamps, P. Schwartz, D. Trebotich, P. Colella, D. Saloner, R. Malladi, *CARS 2004*

Thank you